

Sequent calculi for logics of agency: the deliberative STIT

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STIT Modalities

STIT (see-to-it-that) modalities play a pivotal role in the logic of agency.

They can give formal meaning to various linguistic forms:

- Indicative - *Alice prepares her slides before leaving for the conference.*
- Imperative - *Alice, prepare your slides before leaving for the conference!*
- Subjunctive - *Alice should have prepared her slides before leaving for the conference.*

They can be

- positive
- negative (do otherwise, avoid doing, prevent, refrain, etc.)

STIT Modalities

STIT modalities can be counterfactual modalities

- could have done
- might have done
- should have done

They can occur in the scope of deontic modalities

- oblige (to do something)
- forbid
- permit

They interact with temporal modalities: time of evaluation may refer to a different time of action, e.g.

- duty to apologize
- duty to admonish
- achievement STIT

STIT Modalities

STIT modalities are traditionally defined upon indeterminist frames enriched with agency; semantics builds upon a combination of

- Prior-Thomason-Kripke branching-time semantics
- Kaplan's indexical semantics

The proof theory for these logics has been largely restricted to axiomatic systems.

Both semantics can be approached proof-theoretically in labelled deductive systems, work so far has used *labelled tableaux*:

- Multi-agent deliberative STIT/imagination logic through labelled tableaux, using Belnap's semantics: Wansing (2006)
- STIT/imagination logic through labelled tableaux, using neighbourhood semantics: Wansing & Olkhovikov (2018)

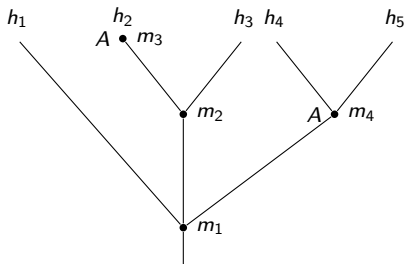
Recent development by van Berkel and Lyon (2019) uses a simplified semantics for atemporal STIT

STIT Modalities

Our aim: Develop systems of sequent calculus that covers in a uniform way *all* the STIT modalities presented by Belnap et al. (2001) (FF) and respect all the desiderata of good proof systems.

We start by treating the deliberative STIT.

Branching time (BT)

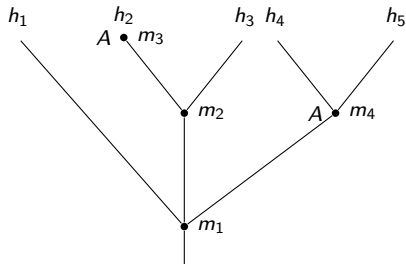


Moments are ordered by a preorder \leq in a treelike structure with

- forward branching (indeterminacy of the future)
- no backward branching (determinacy of the the past)

History is a maximal set of moments lineary ordered by $<$.

Branching time (cont.)



Evaluation of sentences in branching temporal structures - simple example (FF) shows that it cannot be referred to just moments:

Does $m_1 \Vdash Will(A)$ hold?

Not well defined.

Evaluation becomes well defined if performed on moment/history pairs m/h where $m \in h$.

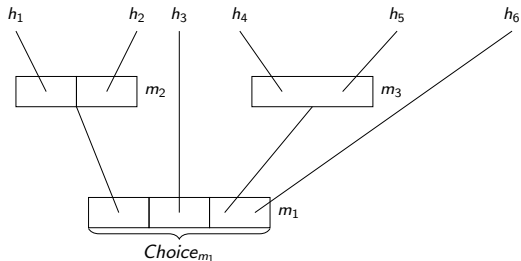
Adding agents and choices

Definition (DSTIT frame)

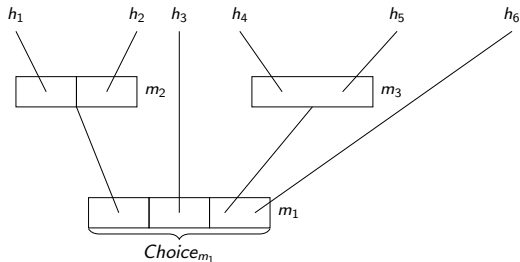
Given a branching temporal frame (T, \leq) , a nonempty set (of agents) $Agent$, a **dstit frame** is obtained by adding

Choice - a function sending any agent/moment-pair (α, m) to a partition H_m of moments passing through m .

Each equivalence class in the partition gives the histories choice-equivalent for α at m .



Adding agents and choices (cont.)



No choice between undivided histories:

If two histories are undivided at m , i.e. there is a future moment that belongs to both, they are choice-equivalent for any agent.

$$\exists m'(m < m' \ \& \ m' \in h \cap h') \rightarrow h \sim_m^\alpha h'$$

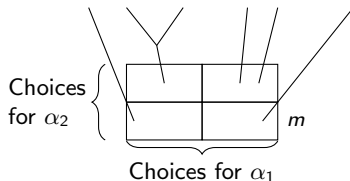
Adding agents and choices (cont.)

Additional assumption in the presence of more than one agent:

- *Independence*

No choice by one agent can make it impossible for another agent to make a simultaneous choice.

So each square of the cartesian product of choices is inhabited by some history:



For each moment m and for a given function f_m such that for each agent α and $f_m(\alpha) \in \text{Choice}(\alpha, m)$, $\bigcap_{\alpha \in \text{Agent}} f_m(\alpha) \neq \emptyset$

$$\text{Diff}(\alpha_1, \dots, \alpha_k) \& m \in h_1 \& \dots \& m \in h_k \rightarrow \exists h. h \sim_m^{\alpha_1} h_1 \& \dots \& h \sim_m^{\alpha_k} h_k$$

From frames to models

Given a dstit frame $(T, \leq, Agent, Choice)$,

Definition (DSTIT model)

A **DSTIT model** is $(T, \leq, Agent, Choice, \mathcal{V})$, where \mathcal{V} is a given valuation function of atomic formulas by sets of moment/history-pairs (*points* for short).

The valuation is extended inductively to dstit-formulas:

$(m, h) \Vdash [i \text{ dstit} : A]$ iff

- 1 $\forall h'. h \sim_m^i h' \rightarrow (m, h') \Vdash A$
- 2 $\exists h'. m \in h' \ \& \ (m, h') \nVdash A$

A formula A is said to be *satisfiable* in this semantics iff there exists a DSTIT model $M = (T, \leq, Agent, Choice, \mathcal{V})$ and a point (m, h) such that $M, (m, h) \Vdash A$.

A formula A is *valid* if it is true at any point in any DSTIT model.

Notation: we write m/h for points (m, h) and $\mathcal{D}^i A$ for $[i \text{ dstit} : A]$

G3DSTIT

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G3DSTIT

These relatively complex truth conditions are transformed into rules of a G3-style labelled sequent calculus with the help of auxiliary modalities:

Definition (Cstit, \Box^i)

$$m/h \Vdash \Box^i A \equiv \forall h' (h' \sim_m^i h \rightarrow m/h' \Vdash A)$$

$$\frac{h' \sim_m^i h, \Gamma \Rightarrow \Delta, m/h' : A}{\Gamma \Rightarrow \Delta, m/h : \Box^i A} R\Box^i, h' \text{ fresh}$$

$$\frac{h' \sim_m^i h, m/h : \Box^i A, m/h' : A, \Gamma \Rightarrow \Delta}{h' \sim_m^i h, m/h : \Box^i A, \Gamma \Rightarrow \Delta} L\Box^i$$

G3DSTIT (cont.)

Two more modalities will be useful, both agent-independent:

Definition (Settled true, \mathcal{S} ; Possible, \mathcal{P})

$$m/h \Vdash \mathcal{S}A \equiv \forall h'(m \in h' \rightarrow m/h' \Vdash A)$$

$$m/h \Vdash \mathcal{P}A \equiv \exists h'(m \in h' \ \& \ m/h' \Vdash A)$$

Their rules follow the patterns of alethic modality:

$$\frac{m \in h', m/h' : A, m/h : \mathcal{S}A, \Gamma \Rightarrow \Delta}{m \in h', m/h : \mathcal{S}A, \Gamma \Rightarrow \Delta} \text{LS}$$

$$\frac{m \in h', m/h' : A, \Gamma \Rightarrow \Delta}{m/h : \mathcal{P}A, \Gamma \Rightarrow \Delta} \text{LP, } h' \text{ fresh}$$

$$\frac{m \in h', \Gamma \Rightarrow \Delta, m/h' : A}{\Gamma \Rightarrow \Delta, m/h : \mathcal{S}A} \text{RS, } h' \text{ fresh}$$

$$\frac{m \in h', \Gamma \Rightarrow \Delta, m/h : \mathcal{P}A, m/h' : A}{m \in h', \Gamma \Rightarrow \Delta, m/h : \mathcal{P}A} \text{RP}$$

G3DSTIT (cont.)

We now introduce the rules for dstit: first we rewrite the truth conditions as rules using the modalities already defined

$$\frac{\Gamma \Rightarrow \Delta, m/h : \Box^i A \quad \Gamma \Rightarrow \Delta, m/h : \neg SA}{\Gamma \Rightarrow \Delta, m/h : \mathcal{D}^i A} \overline{RD^i}$$

$$\frac{m/h : \Box^i A, m/h : \neg SA, \Gamma \Rightarrow \Delta}{m/h : \mathcal{D}^i A, \Gamma \Rightarrow \Delta} \overline{LD^i}, h' \text{ fresh}$$

Second, we eliminate the negation by the use of the classical symmetry of the calculus:

$$\frac{\Gamma \Rightarrow \Delta, m/h : \Box^i A \quad m/h : SA, \Gamma \Rightarrow \Delta}{\Gamma \Rightarrow \Delta, m/h : \mathcal{D}^i A} RD^i$$

$$\frac{m/h : \Box^i A, \Gamma \Rightarrow \Delta, m/h : SA}{m/h : \mathcal{D}^i A, \Gamma \Rightarrow \Delta} LD^i$$

G3DSTIT (cont.)

We also have to make explicit the rules that correspond to the properties of the equivalence relation between histories as well as equality of agents.

As usual, the equivalence relation can be given by just two rules, Reflexivity and Euclidean transitivity:

$$\frac{h \sim_m^i h, \Gamma \Rightarrow \Delta}{\Gamma \Rightarrow \Delta} \text{Refl}_{\sim_m^i} \quad \frac{h_2 \sim_m^i h_3, h_1 \sim_m^i h_2, h_1 \sim_m^i h_3, \Gamma \Rightarrow \Delta}{h_1 \sim_m^i h_2, h_1 \sim_m^i h_3, \Gamma \Rightarrow \Delta} \text{Etrans}_{\sim_m^i}$$

$$\frac{i = i, \Gamma \Rightarrow \Delta}{\Gamma \Rightarrow \Delta} \text{Refl}_= \quad \frac{j = k, i = j, i = k, \Gamma \Rightarrow \Delta}{i = j, i = k, \Gamma \Rightarrow \Delta} \text{Etrans}_=$$

$$\frac{i = j, \text{At}(i), \text{At}(j), \Gamma \Rightarrow \Delta}{i = j, \text{At}(i), \Gamma \Rightarrow \Delta} \text{Repl}_{\text{At}} \quad \frac{m \in h, h \sim_m^i h', \Gamma \Rightarrow \Delta}{h \sim_m^i h', \Gamma \Rightarrow \Delta} \text{WD}$$

G3DSTIT (cont.)

We now account for the independence of agents. We first define the rule for difference of agents:

$$\frac{}{i \neq j, i = j, \Gamma \Rightarrow \Delta} \neq \frac{\{i_l \neq i_m\}_{1 \leq l < m \leq k}, \Gamma \Rightarrow \Delta}{\text{Diff}(i_1, \dots, i_k), \Gamma \Rightarrow \Delta} \text{Diff}_k$$

(where $i \neq j \supset \neg i = j$) and then introduce the Independence of agents rule (first attempt):

$$\frac{h \sim_m^{i_1} h_1, \dots, h \sim_m^{i_k} h_k, \text{Diff}(i_1, \dots, i_k), m \in h_1, \dots, m \in h_k, \Gamma \Rightarrow \Delta}{\text{Diff}(i_1, \dots, i_k), m \in h_1, \dots, m \in h_k, \Gamma \Rightarrow \Delta} \overline{\text{Ind}_k}, h \text{ fresh}$$

For k agents and k histories, there is a history compatible with any of the former choosing any of the latter.

G3DSTIT (cont.)

Choice-equivalence relation only features in rules with agent-relative formulas. So, we can limit the rule to agent-relative formulas as well.

Intuitively, not all the agents need to be choosing, and they need not be choosing among all the histories. We can limit the rule to only those agents that are choosing and only those histories they are choosing from.

This gives the final, parametrized version of the rule:

$$\frac{h \sim_m^{i_1} h_1, \dots, h \sim_m^{i_n} h_n, \text{Diff}(i_1, \dots, i_k), m \in h_1, \dots, m \in h_n, h_1 \sim_m^{i_1} h'_1, \dots, h_n \sim_m^{i_n} h'_n, \Gamma \Rightarrow \Delta}{\text{Diff}(i_1, \dots, i_k), m \in h_1, \dots, m \in h_n, h_1 \sim_m^{i_1} h'_1, \dots, h_n \sim_m^{i_n} h'_n, \Gamma \Rightarrow \Delta} \text{Ind}_k, h \text{ fresh}$$

There is a history compatible with any agent that is choosing among histories choosing any of the histories he/she is choosing from.

G3DSTIT (cont.)

Properties of $BT+AC$ frames can be formulated as rules that follow the regular rule scheme.

However, all the logical rules, when applied root-first, may modify only histories, and the moment of evaluation remains unchanged.

It follows that the relational rules give a conservative extension, so we will omit them from our calculus.

Results

- ① STIT
- ② G3DSTIT
- ③ **Results**
- ④ Applications
- ⑤ Future work

Axiomatization

AI_{A_k}: If $\text{Diff}(i_1, \dots, i_k)$, then

$$\mathcal{P}\Box^{i_1} A_1 \& \dots \& \mathcal{P}\Box^{i_k} A_k \supset \mathcal{P}(\Box^{i_1} A_1 \& \dots \& \Box^{i_k} A_k)$$

For “simplicity”, we prove **AI_{A₂}**. The generalization to k agents is straightforward.

(1)

$$\frac{\frac{m/h_4 : A_1, h_4 \sim_m^{a_1} h_1, h_4 \sim_m^{a_1} h_3, \dots, \text{Diff}(a_1, a_2), m/h_1 : \Box^{a_1} A_1, m/h_2 : \Box^{a_2} A_2 \Rightarrow \dots, m/h_4 : A_1}{h_4 \sim_m^{a_1} h_1, h_4 \sim_m^{a_1} h_3, h_3 \sim_m^{a_1} h_1, \dots, \text{Diff}(a_1, a_2), m/h_1 : \Box^{a_1} A_1, m/h_2 : \Box^{a_2} A_2 \Rightarrow \dots, m/h_4 : A_1} \quad L\Box^1}{h_4 \sim_m^{a_1} h_3, h_3 \sim_m^{a_1} h_1, \dots, \text{Diff}(a_1, a_2), m/h_1 : \Box^{a_1} A_1, m/h_2 : \Box^{a_2} A_2 \Rightarrow \dots, m/h_4 : A_1} \quad R\Box^1}{h_3 \sim_m^{a_1} h_1, \dots, \text{Diff}(a_1, a_2), m/h_1 : \Box^{a_1} A_1, m/h_2 : \Box^{a_2} A_2 \Rightarrow \dots, m/h_3 : \Box^{a_1} A_1} \quad ETrans_{\sim_m^{a_1}}$$

(1)

(2)

$$\frac{\frac{m \in h_3, h_3 \sim_m^{a_1} h_1, \dots, \text{Diff}(a_1, a_2), m/h_1 : \Box^{a_1} A_1, m/h_2 : \Box^{a_2} A_2 \Rightarrow \dots, m/h_3 : \Box^{a_1} A_1 \& \Box^{a_2} A_2}{m \in h_3, h_3 \sim_m^{a_1} h_1, \dots, \text{Diff}(a_1, a_2), m/h_1 : \Box^{a_1} A_1, m/h_2 : \Box^{a_2} A_2 \Rightarrow m/h : \mathcal{P}(\Box^{a_1} A_1 \& \Box^{a_2} A_2)} \quad R\&}{h_3 \sim_m^{a_1} h_1, h_3 \sim_m^{a_2} h_2, h_1 \sim_m^{a_1} h_1, h_2 \sim_m^{a_2} h_2, \text{Diff}(a_1, a_2), m/h_1 : \Box^{a_1} A_1, m/h_2 : \Box^{a_2} A_2 \Rightarrow m/h : \mathcal{P}(\Box^{a_1} A_1 \& \Box^{a_2} A_2)} \quad RP}{h_1 \sim_m^{a_1} h_1, h_2 \sim_m^{a_2} h_2, \text{Diff}(a_1, a_2), m/h_1 : \Box^{a_1} A_1, m/h_2 : \Box^{a_2} A_2 \Rightarrow m/h : \mathcal{P}(\Box^{a_1} A_1 \& \Box^{a_2} A_2)} \quad WD}{\frac{\text{Diff}(a_1, a_2), m/h_1 : \Box^{a_1} A_1, m/h_2 : \Box^{a_2} A_2 \Rightarrow m/h : \mathcal{P}(\Box^{a_1} A_1 \& \Box^{a_2} A_2)}{\text{Diff}(a_1, a_2), m/h : \mathcal{P}\Box^{a_1} A_1, m/h : \mathcal{P}\Box^{a_2} A_2 \Rightarrow m/h : \mathcal{P}(\Box^{a_1} A_1 \& \Box^{a_2} A_2)} \quad L\mathcal{P}, L\mathcal{P}} \quad Ind_2, Ref_{\sim_m^{a_1}}, Ref_{\sim_m^{a_2}}$$

(2) is similar to (1).

Structural properties

All of the following hold of G3DSTIT:

- Derivability of initial sequents of the form $m/h : A, \Gamma \Rightarrow \Delta, m/h : A$ where A is an arbitrary formula in the DSTIT language
- Height-preserving substitution on moments/histories
- Height-preserving admissibility of weakening
- Height-preserving invertibility of all the rules
- Height-preserving admissibility of contraction
- Admissibility of cut

Because of course they do.

A suitable notion of weight of formulas is needed. Importantly:

- $w(\Box^\alpha A) = w(\mathcal{S}A) = w(\mathcal{P}A) = w(A) + 1$
- $w(\mathcal{D}^\alpha A) = w(A) + 2$

The weight reflects the way we have unfolded the rule of the dstit operator using additional modalities, and guarantees that each time we use a rule for such modalities the weight of active formulas is less than the weight of principal formulas.

Structural properties (cont.)

$$\frac{\frac{m \in h', \Gamma \Rightarrow \Delta, m/h : \Box^i A \quad m \in h', m/h' : A, \Gamma \Rightarrow \Delta}{m \in h', \Gamma \Rightarrow \Delta, m/h : \mathcal{D}^i A} \quad R\mathcal{D}^i \quad \frac{m \in h'', m/h : \Box^i A, \Gamma \Rightarrow \Delta, m/h'' : A}{m/h : \mathcal{D}^i A, \Gamma \Rightarrow \Delta} \quad LD^i}{m \in h', \Gamma \Rightarrow \Delta} \quad Cut$$

This transformed into:

(1)

$$\frac{m \in h', m/h : \Box^i : A, \Gamma \Rightarrow \Delta, m/h' : A \quad \frac{m \in h', m/h' : A, \Gamma \Rightarrow \Delta, m/h : \mathcal{D}^i A \quad m/h : \mathcal{D}^i A, \Gamma \Rightarrow \Delta}{m \in h', m/h' : A, \Gamma \Rightarrow \Delta} \quad Cut_2}{m \in h', m/h : \Box^i : A, \Gamma \Rightarrow \Delta} \quad Cut_3$$

$$\frac{\frac{m \in h', \Gamma \Rightarrow \Delta, m/h : \Box^i A, m/h : \mathcal{D}^i A \quad m/h : \mathcal{D}^i A, \Gamma \Rightarrow \Delta}{m \in h', \Gamma \Rightarrow \Delta, m/h : \Box^i A} \quad Cut_1 \quad (1)}{m \in h', \Gamma \Rightarrow \Delta} \quad Cut_4$$

where Cut_1 and Cut_2 are of lesser height, and Cut_3 and Cut_4 are of lesser weight.

Decidability

To obtain a decidable system we adopt the following axiom. As shown in FF (p. 437), the validity of APC_n in DSTIT frames is equivalent to the requirement that the agent i has at most n choices at any moment, i.e. there are at most n partitions given by \sim_m^i .

APC_n :

$$\mathcal{P}\Box^i A_1 \ \& \ \mathcal{P}(\neg A_1 \ \& \ \Box^i A_2) \ \& \ \dots \ \& \ \mathcal{P}(\neg A_1 \ \& \ \dots \ \& \ \neg A_{n-1} \ \& \ \Box^i A_n) \supset A_1 \vee \dots \vee A_n$$

The corresponding rule extends our calculus **G3DSTIT** and the resulting calculus will be denoted by **G3Ldm_n**:

$$\frac{h_1 \sim_m^i h_2, m \in h_1, \dots, h_{n+1}, \Gamma \Rightarrow \Delta \quad \dots \quad h_n \sim_m^i h_{n+1}, m \in h_1, \dots, h_{n+1}, \Gamma \Rightarrow \Delta}{m \in h_1, \dots, m \in h_{n+1}, \Gamma \Rightarrow \Delta} \text{APC}_n$$

Decidability (cont.)

We present a direct proof of decidability through a bound on proof search.

We provide a decision procedure for **G3Ldm_n**, by showing that proof search always terminates in a finite number of steps. To that end, we employ the notion of saturation in the standard way.

Definition (Saturation)

Let $\mathcal{B} = \{\Gamma_n \Rightarrow \Delta_n\}$ be a (finite or infinite) branch in proof search for $\Gamma \Rightarrow \Delta$, and let $\Gamma^* = \bigcup \Gamma_n$, $\Delta^* = \bigcup \Delta_n$.

- (\mathcal{LD}^i): If $m/h : \mathcal{D}^i A$ is in Γ^* , then $m/h : \Box^i A$ is in Γ^* and $m/h : SA$ is in Δ^* .
- (Ind_k): If $\text{Diff}(a_1 \dots a_k)$ is in Γ^* , and for any a_i and a_j , $1 \leq i < j \leq k$, $h_i \sim_m^i h'_i$ and $h_j \sim_m^j h'_j$ are in Γ^* , then for some history h , $h \sim_m^i h_i$ and $h \sim_m^j h_j$ are also in Γ^* .

We call the branch \mathcal{B} saturated w.r.t. an application of a rule if the corresponding condition holds, and saturated simpliciter if it is saturated w.r.t. all the rules.

Decidability (cont.)

We can now build (root-first) a proof-search tree for a sequent $\Rightarrow m/h_0 : A_0$.

Rule R is not applied to a sequent $\Gamma_i \Rightarrow \Delta_i$ if the branch \mathcal{B} down to $\Rightarrow m/h_0 : A_0$ is saturated w.r.t. R .

We focus on the rule Ind^k , which is applied in a special sequence.

Definition (Independence point)

If m/h' is a point with a fresh history h' generated by an application of rule Ind_k and $m \in h_1 \dots m \in h_n$ are among the principal formulas of the rule, we call m/h' an **independence point**.

We show a crucial lemma.

Decidability (cont.)

Lemma

The number of independence points in \mathcal{B} is finite.

PROOF. By showing that the generation of new independence points via the applications of Ind_k terminates.

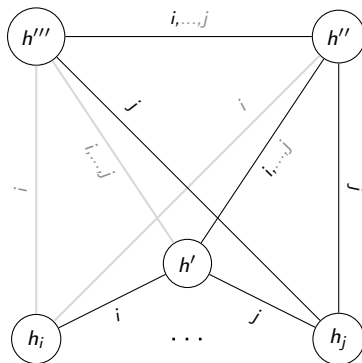
Intuitively, the main idea is that each application of Ind_k generates a new independence point, which inherits all the choice equivalence relations.

So, now we need to apply Ind_k to it as well.

Once we have generated three independence point, each of which are connected to the other two with all the inherited choice-equivalence relation, the process stops due to the saturation criterion.

Decidability (cont.)

Let's draw a picture:



Decidability (cont.)

Now take the lowest $\Gamma_i \Rightarrow \Delta_i$ such that it is the lower sequent of an Ind_k rule. Let S be a set of all maximal sets of points in Γ_i that Ind_k can be applied to.

Once the saturation criterion is met, by the above procedure and ending at a sequent $\Gamma_n \Rightarrow \Delta_n$, for each element of S , \mathcal{B} up to $\Gamma_n \Rightarrow \Delta_n$ is saturated w.r.t Ind_k .

There are three cases to check:

- ① two histories, say h_i and h_j (with agents a_i and a_j) from different elements:
 - by the definition of S , there is an element that contains both h_i and h_j . Therefore, the saturation criterion is met.
- ② a history, say h_i , and an independence point h'_j from different elements:
 - there is an h_j such that $h'_j \sim_m^{a_j} h_j$, and point (1) applies.
- ③ two independence points, say h'_i and h'_j from different elements:
 - point (2) applies twice.

Same for any subsequent sequent $\Gamma_j \Rightarrow \Delta_j$.

Completeness

G3Ldm_n is complete with respect to the semantics of DSTIT.

PROOF. By generating a countermodel from a saturated branch. Given a saturated branch \mathcal{B} in a search for a proof of the sequent $\Gamma \Rightarrow \Delta$, we generate a DSTIT countermodel \mathcal{M} that makes all the formulas in Γ^* true and all formulas in Δ^* false.

To illustrate,

Assume $m/h : \mathcal{D}^{a_i} A$ is in Γ^* .

Then, by the saturation criterion,

$m/h : \Box^{a_i} A$ is in Γ^* and $m/h : \mathcal{S}A$ is in Δ^* .

So, by the inductive hypothesis,

- ① $\mathcal{M}, (m, h) \Vdash \Box^{a_i} A$
- ② $\mathcal{M}, (m, h) \not\Vdash \mathcal{S}A$

It follows that $\mathcal{M}, (m, h) \Vdash \mathcal{D}^{a_i} A$

Applications

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Interdefinability

As noted in (FF, p.298), each of \Box^i , \mathcal{D}^i and S is definable using the other two.

- $\Box^i A \supset \supset \mathcal{D}^i A \vee SA$
- $\mathcal{D}^i A \supset \supset \Box^i A \ \& \ \neg SA$
- $SA \supset \supset \Box^i A \ \& \ \neg \mathcal{D}^i A$

Meta-Agency (cont.)

We can treat nested STIT modalities and agents can be different for each of the nested modalities, that is, we have individual multiple agency.

Is it possible that an agent sees to it that another agent sees to it that A ?

Hint: we are considering *independent* agents.

(FF, p. 274) gives a semantic argument to show that this is impossible for the achievement STIT.

We can give a proof-theoretic argument to show that assuming $m/h : \mathcal{D}^\alpha \mathcal{D}^\beta A$ leads to a contradiction (independently of the form of A). In our system this is shown by a derivation of

$$m/h : \mathcal{D}^\alpha (\mathcal{D}^\beta A) \Rightarrow$$

Meta-Agency (cont.)

Lemma

The following sequent is provable in G3-dstit:

$$h_1 \sim_m^\alpha h_2, m/h_1 : \mathcal{D}^\alpha A \Rightarrow m/h_2 : \mathcal{D}^\alpha A.$$

Proof.

$$\frac{
 \frac{
 \frac{
 h_1 \sim_m^\alpha h_4, \dots, m \in h_3, m/h_4 : A, m/h_1 : \Box^\alpha A \Rightarrow m/h_4 : A, m/h_3 : A
 }{
 h_1 \sim_m^\alpha h_4, \dots, m \in h_3, m/h_1 : \Box^\alpha A \Rightarrow m/h_4 : A, m/h_3 : A
 }
 L\Box^\alpha
 }{
 h_1 \sim_m^\alpha h_2, h_4 \sim_m^\alpha h_2, m \in h_3, m/h_1 : \Box^\alpha A \Rightarrow m/h_4 : A, m/h_3 : A
 }
 Etrans_{\sim_m^\alpha}
 }{
 h_1 \sim_m^\alpha h_2, m \in h_3, m/h_1 : \Box^\alpha A \Rightarrow m/h_2 : \Box^\alpha A, m/h_3 : A
 }
 R\Box^\alpha
 \quad \dots m/h_3 : A \Rightarrow m/h_3 : A \dots
 }{
 \frac{
 h_1 \sim_m^\alpha h_2, m \in h_3, m/h_1 : \Box^\alpha A \Rightarrow m/h_2 : \mathcal{D}^\alpha A, m/h_3 : A
 }{
 h_1 \sim_m^\alpha h_2, m/h_1 : \mathcal{D}^\alpha A \Rightarrow m/h_2 : \mathcal{D}^\alpha A
 }
 LD^\alpha
 }
 RD^\alpha$$



Meta-Agency (cont.)

$$\begin{array}{c}
 \text{Lemma} \\
 \hline
 h_3 \sim_m^\alpha h_1, h_3 \sim_m^\beta h_2, m/h_3 : \mathcal{D}^\beta A, m/h_1 : \square^\alpha(\mathcal{D}^\beta A), m \in h_2 \Rightarrow m/h_2 : \mathcal{D}^\beta A \\
 \hline
 h_3 \sim_m^\alpha h_1, h_3 \sim_m^\beta h_2, m_1/h : \square^\alpha(\mathcal{D}^\beta A), m \in h_2 \Rightarrow m/h_2 : \mathcal{D}^\beta A \quad \text{Ind} \\
 \hline
 m/h_1 : \square^\alpha(\mathcal{D}^\beta A), m \in h_2 \Rightarrow m/h_2 : \mathcal{D}^\beta A \quad \text{LD}^\alpha \\
 \hline
 m/h_1 : \mathcal{D}^\alpha(\mathcal{D}^\beta A) \Rightarrow
 \end{array}$$

We have therefore proved **impossibility of delegation**

Meta-agency (cont.)

We can likewise show it is impossible to *prevent* somebody from doing something:

$$\begin{array}{c}
 \vdots \\
 \hline
 \dots, h^i \sim_m^b h', \dots, m/h' : \Box^b A \Rightarrow \dots, m/h^i : \Box^b A \quad \text{Lemma} \quad \frac{m/h'' : A, \dots \Rightarrow m/h'' : A, \dots}{m/h^i : SA, m \in h'', \dots \Rightarrow m/h'' : A, \dots} \quad LS \\
 \hline
 \frac{h^i \sim_m^a h, h^i \sim_m^b h', h \sim_m^a h, h' \sim_m^b h', m \in h', h'', m/h : \Box^a \neg \mathcal{D}^b A, m/h' : \Box^b A \Rightarrow m/h'' : A, m/h^i : \mathcal{D}^b A}{h^i \sim_m^a h, h^i \sim_m^b h', h \sim_m^a h, h' \sim_m^b h', m \in h', h'', m/h : \Box^a \neg \mathcal{D}^b A, m/h' : \neg \mathcal{D}^b A, m/h' : \Box^b A \Rightarrow m/h'' : A} \quad RD^b \\
 \hline
 \frac{h^i \sim_m^a h, h^i \sim_m^b h', h \sim_m^a h, h' \sim_m^b h', m \in h', h'', m/h : \Box^a \neg \mathcal{D}^b A, m/h' : \Box^b A \Rightarrow m/h'' : A}{h^i \sim_m^a h, h^i \sim_m^b h', h \sim_m^a h, h' \sim_m^b h', m \in h', h'', m/h : \Box^a \neg \mathcal{D}^b A, m/h' : \Box^b A \Rightarrow m/h'' : A} \quad L\Box^a \\
 \hline
 \frac{h^i \sim_m^a h, h^i \sim_m^b h', h \sim_m^a h, h' \sim_m^b h', m \in h', h'', m/h : \Box^a \neg \mathcal{D}^b A, m/h' : \Box^b A \Rightarrow m/h'' : A}{h \sim_m^a h, h' \sim_m^b h', m \in h', h'', m/h : \Box^a \neg \mathcal{D}^b A, m/h' : \Box^b A \Rightarrow m/h'' : A} \quad Ind_2 \\
 \hline
 \frac{h \sim_m^a h, h' \sim_m^b h', m \in h', h'', m/h : \Box^a \neg \mathcal{D}^b A, m/h' : \Box^b A \Rightarrow m/h'' : A}{h \sim_m^a h, h' \sim_m^b h', m \in h', m/h : \Box^a \neg \mathcal{D}^b A, m/h' : \mathcal{D}^b A \Rightarrow} \quad LD^b, RS \\
 \hline
 \frac{h \sim_m^a h, h' \sim_m^b h', m \in h', m/h : \Box^a \neg \mathcal{D}^b A, m/h' : \mathcal{D}^b A \Rightarrow}{h \sim_m^a h, h' \sim_m^b h', m \in h', m/h : \Box^a \neg \mathcal{D}^b A \Rightarrow m/h' : \neg \mathcal{D}^b A} \quad R\neg \\
 \hline
 \frac{m \in h', m/h : \Box^a \neg \mathcal{D}^b A \Rightarrow m/h' : \neg \mathcal{D}^b A}{m/h : \mathcal{D}^a \neg \mathcal{D}^b A \Rightarrow} \quad Refl \\
 \hline
 \frac{m \in h', m/h : \Box^a \neg \mathcal{D}^b A \Rightarrow m/h' : \neg \mathcal{D}^b A}{m/h : \mathcal{D}^a \neg \mathcal{D}^b A \Rightarrow} \quad LD^a, RS
 \end{array}$$

Refraining

G.H. von Wright, 1963:

Events as ordered pairs of states of affairs,

- 1 *initial state* p , temporally preceding the
- 2 *end-state* q .

An event itself, $p T q$, is a transition from the former to the latter.

An *act* is the bringing about of an event by an agent, written as $d(p T q)$.

Condition for doing $d(\sim p T p)$ -bringing about p - is that p does not happen “independently of the action of the agent” - clear connection to STIT.

Refraining (cont.)

The “correlative” of doing is to refrain from doing (von Wright - ‘forbear’).

Not simply not doing an action!

To forbear p , $f(\sim p T p)$, is to be able to do it, but not do it.

In our notation it would be understood as:

$$Ref^i A \equiv_{def.} \mathcal{P}D^i A \ \& \ \neg D^i A$$

Acts and forbearances are modes of action - forbearance also does not come about independently of an agent.

In (FF) refraining is analysed by using embedded modalities. Noting that refraining itself is a mode of doing, the definition becomes

$$Ref^i A \equiv_{def.} D^i \neg D^i A$$

We can show that the two accounts are equivalent (cf. FF, p.438):

$$D^i \neg D^i A \equiv \mathcal{P}D^i A \ \& \ \neg D^i A$$

Refraining (cont.)

Under this interpretation, it holds for DSTIT that doing is equivalent to refraining from refraining (FF, p. 50, 439):

$$\text{(Refref): } \mathcal{D}^i A \equiv \mathcal{D}^i \neg \mathcal{D}^i \neg \mathcal{D}^i A$$

We can likewise show this holds for DSTIT. This is expressed in our system as the equivalence, meaning the sequents in both directions,

$$m/h : \mathcal{D}^i A \Leftrightarrow m/h : \mathcal{D}^i \neg \mathcal{D}^i \neg \mathcal{D}^i A$$

Refraining (cont.)

(1)

$$\frac{\frac{m \in h, h_1, m/h : \Box^j A \Rightarrow m/h_1 : A, m/h : \Box^j A \quad m \in h, h_1, m/h : \Box^j A, m/h_1 : A \Rightarrow m/h_1 : A}{m \in h, h_1, m/h : \Box^j A \Rightarrow m/h_1 : A, m/h : \Box^j A} \text{RD}^j}{m \in h, h_1, m/h : \neg \Box^j A, m/h : \Box^j A \Rightarrow m/h_1 : A} \text{L}\neg$$

(2)

$$\frac{\frac{\frac{h_2 \sim_m^j h_1, m/h_2 : \Box^j A \Rightarrow m/h_1 : A}{\vdots} \quad \frac{h_2 \sim_m^j h_1, m \in h, h_1, m/h : \Box^j A, m/h_2 : \Box^j A \Rightarrow m/h_1 : A, m/h_1 : \mathcal{D}^j \neg \mathcal{D}^j A, m/h_3 : A}{h_2 \sim_m^j h_1, m \in h, h_1, m/h : \Box^j A, m/h_2 : \mathcal{D}^j A \Rightarrow m/h_1 : A, m/h_1 : \mathcal{D}^j \neg \mathcal{D}^j A} \text{LD}^j}{h_2 \sim_m^j h_1, m \in h, h_1, m/h : \Box^j A \Rightarrow m/h_1 : A, m/h_1 : \mathcal{D}^j \neg \mathcal{D}^j A, m/h_2 : \neg \mathcal{D}^j A} \text{R}\neg}{\frac{m \in h, h_1, m/h : \Box^j A \Rightarrow m/h_1 : A, m/h_1 : \mathcal{D}^j \neg \mathcal{D}^j A, m/h_1 : \Box^j \neg \mathcal{D}^j A}{m \in h, h_1, m/h : \Box^j A \Rightarrow m/h_1 : A, m/h_1 : \mathcal{D}^j \neg \mathcal{D}^j A} \text{R}\Box^j} \text{(1)} \quad \text{RD}^j}{\frac{\frac{m \in h, h_1, m/h : \Box^j A \Rightarrow m/h_1 : A, m/h_1 : \mathcal{D}^j \neg \mathcal{D}^j A}{m \in h_1, m/h : \Box^j A \Rightarrow m/h_1 : A, m/h_1 : \mathcal{D}^j \neg \mathcal{D}^j A} \text{WD}}{m \in h_1, m/h : \Box^j A, m/h_1 : \neg \mathcal{D}^j \neg \mathcal{D}^j A \Rightarrow m/h_1 : A} \text{L}\neg} \text{WD}}{\frac{m \in h_1, h_3, \dots, m/h : \Box^j A, \dots \Rightarrow \dots, m/h : \Box^j A \quad m \in h_1, h_3, \dots, m/h_1 : A, \dots \Rightarrow m/h_1 : A, \dots}{h_2 \sim_m^j h, m \in h_1, h_3, m/h : \Box^j A, m/h_2 : \Box^j \neg \mathcal{D}^j A, m/h_3 : \Box^j A \Rightarrow m/h_1 : A, m/h : \mathcal{D}^j A, m/h_4 : A} \text{RD}^j}{\frac{h_2 \sim_m^j h, m \in h_1, h_3, m/h : \Box^j A, m/h_2 : \Box^j \neg \mathcal{D}^j A, m/h_3 : \mathcal{D}^j A \Rightarrow m/h_1 : A, m/h : \mathcal{D}^j A}{h_2 \sim_m^j h, m \in h_1, h_3, m/h : \Box^j A, m/h_2 : \Box^j \neg \mathcal{D}^j A, m/h : \neg \mathcal{D}^j A \Rightarrow m/h_1 : A, m/h_3 : \neg \mathcal{D}^j A} \text{LD}^j} \text{L}\neg, \text{R}\neg}{\frac{h_2 \sim_m^j h, m \in h_1, h_3, m/h : \Box^j A, m/h_2 : \Box^j \neg \mathcal{D}^j A \Rightarrow m/h_1 : A, m/h_3 : \neg \mathcal{D}^j A}{h_2 \sim_m^j h, m \in h_1, m/h : \Box^j A, m/h_2 : \mathcal{D}^j \neg \mathcal{D}^j A \Rightarrow m/h_1 : A} \text{LD}^j} \text{L}\Box^j}{\frac{h_2 \sim_m^j h, m \in h_1, m/h : \Box^j A \Rightarrow m/h_1 : A, m/h_2 : \neg \mathcal{D}^j \neg \mathcal{D}^j A}{m \in h_1, m/h : \Box^j A \Rightarrow m/h_1 : A, m/h : \Box^j \neg \mathcal{D}^j \neg \mathcal{D}^j A} \text{R}\neg} \text{R}\Box^j} \text{(2)} \quad \text{RD}^j}{\frac{m \in h_1, m/h : \Box^j A \Rightarrow m/h_1 : A, m/h : \mathcal{D}^j \neg \mathcal{D}^j \neg \mathcal{D}^j A}{m/h : \mathcal{D}^j A \Rightarrow m/h : \mathcal{D}^j \neg \mathcal{D}^j \neg \mathcal{D}^j A} \text{LD}^j} \text{LD}^j$$

Future work

- Achievement STIT
- Intuitionistic version of DSTIT (done)
- Deontic expansion

Thank you!