Automating Agential Reasoning: Proof-Calculi and Syntactic Decidability for (deontic) STIT Logics

Kees van Berkel and Tim Lyon

Institut für Logic and Computation, TU Wien, Vienna, Austria

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logics LOGICAL METHODS IN COMPUTER SCIENCE

FWF Der Wissenschaftsfonds.

WWTF WIENER WISSENSCHAFTS-, FORSCHUNGS- UND TECHNOLOGIEFONDS

Introduction (1/3)

Autonomous systems are more and more integrated in our lives

- They are developed to assist in and take over **human tasks**
- Human acting is inevitably connected to **moral and legal problems**

Problem

These systems must generate **normatively acceptable decisions**

Solution

Development of formal frameworks for **normative choice-making**:

- The **agency logic** of STIT

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STIT Logic ('Seeing To It That') reasoning about **choices of agents**:

- Applications in **legal and deontic reasoning**:
  1. Legal culpability; e.g. [JURIX15]
  2. Legal contracts; e.g. [DEON18]
  3. Utilitarian obligations; e.g. [AiML05] etc...

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Temporal Deontic STIT logic: **open question** since [BelnapPerlof90s]

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Automated reasoning tools for (deontic) STIT logics are **still lacking**:

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2. Provide sequent-style calculi for (deontic) STIT logics
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(**NB.** Yes, this is a long-term project!)

Outline of this talk

1. Introduction
2. STIT in a nutshell
3. Temporal Deontic STIT Logic
4. Calculi for STIT logics
5. Future work

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Semantically, choice **indeterminism** motivates **branching-time** frames

STIT in a nutshell (2/6)

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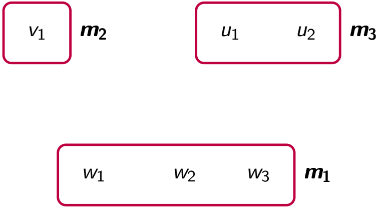
Branching-Time Structures [Lorini13]:

1. **Moments** as sets of worlds
2. Trees as orderings of moments
   - Branching to the future
   - Linear to the past
   - Irreflexive moments

NB. Moments are **equivalence classes**



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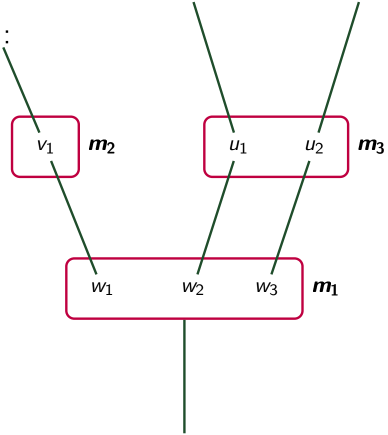

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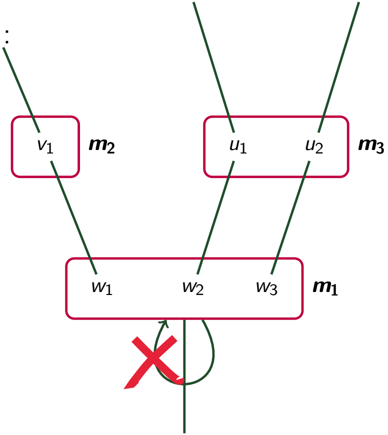

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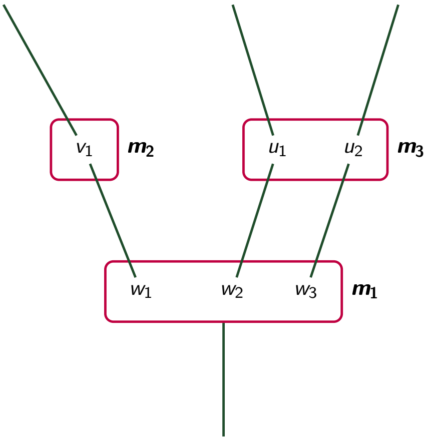
Single-agent STIT:

1. Branching-time frames



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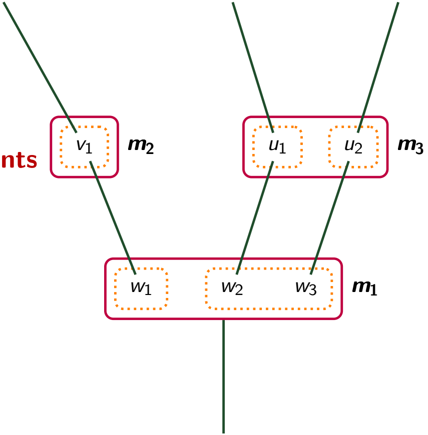

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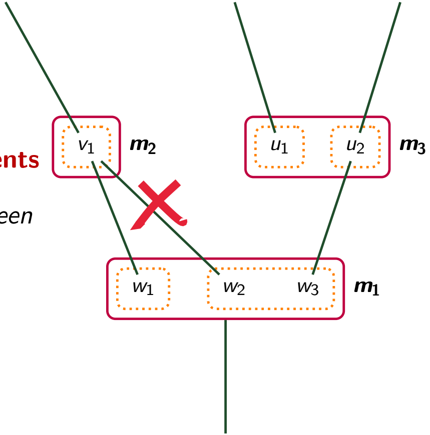
3. **Restriction (1)**: *No Choice Between Undivided "Histories"*



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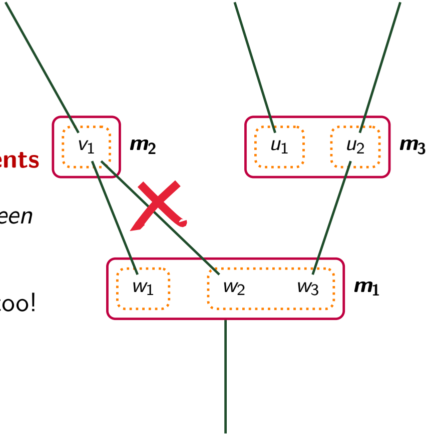

STIT in a nutshell (4/6)

Multi-agent STIT (example):

1. Two agents with two choices:

   Agent 1: $\{w_1, w_2\}, \{w_3, w_4\}$

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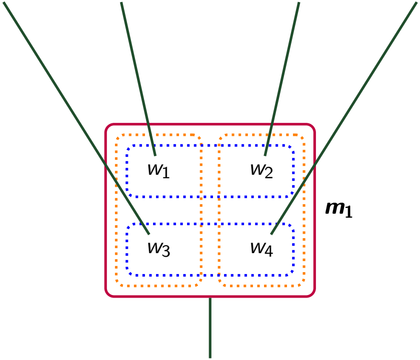

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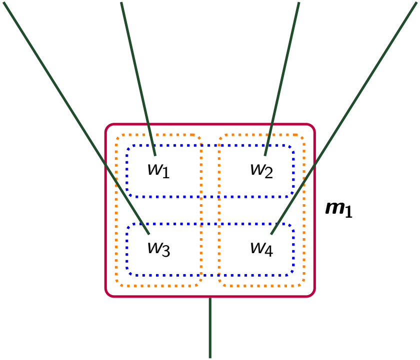

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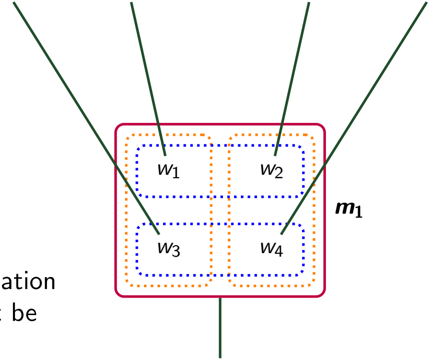
2. **Restriction (2)**:
   Independence of Agents.

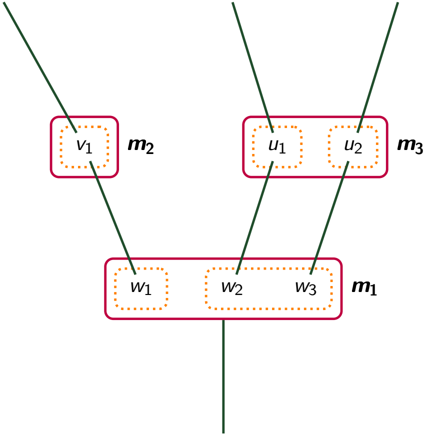
   "The intersection of any combination of different agents' choices must be *non-empty*."



STIT in a nutshell (5/6)

Semantics of (temporal) STIT:

1. Truth is at worlds within moments



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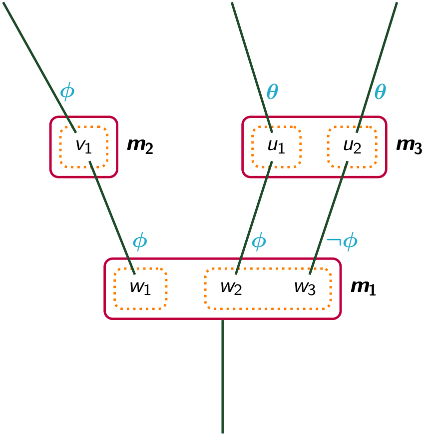

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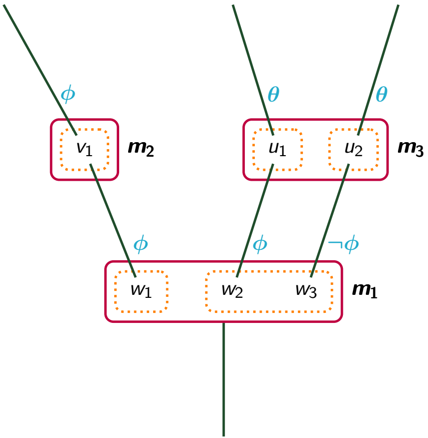

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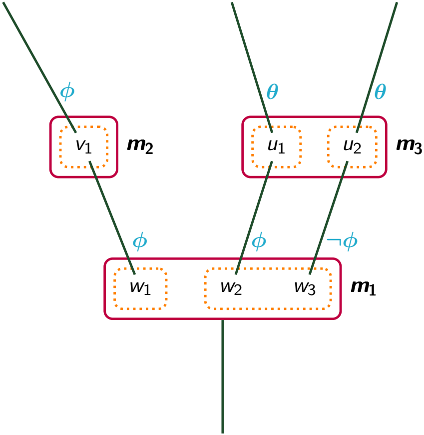

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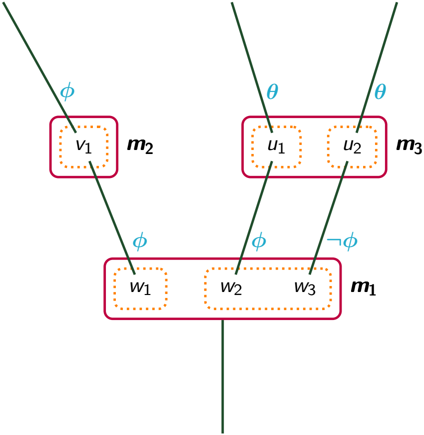

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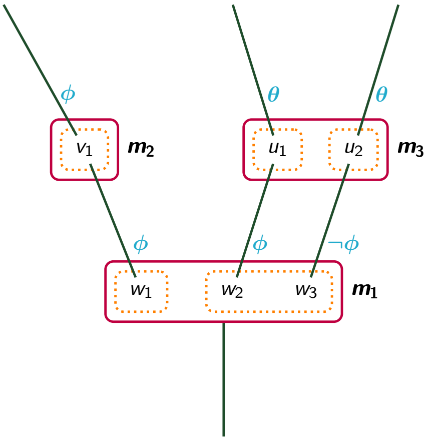

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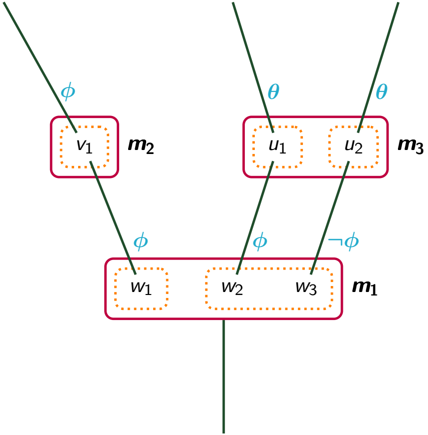

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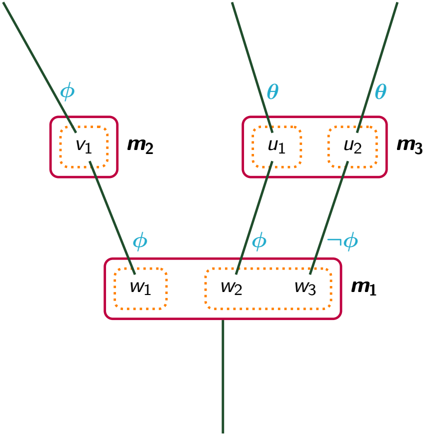
6. Is it possible for the agent at m_1 to see to it that in the future θ holds?



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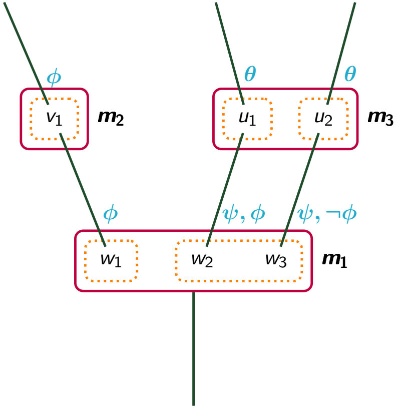
Semantics of Utilitarian STIT [Horty01]:

1. Utility functions

   Assign naturals, reals, binary...

   Fixed for histories, worlds...



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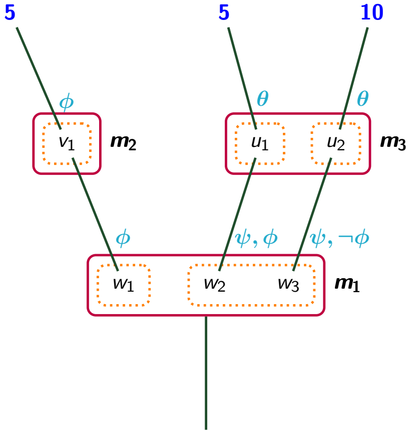

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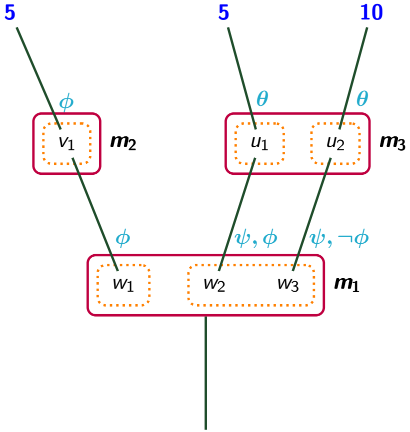

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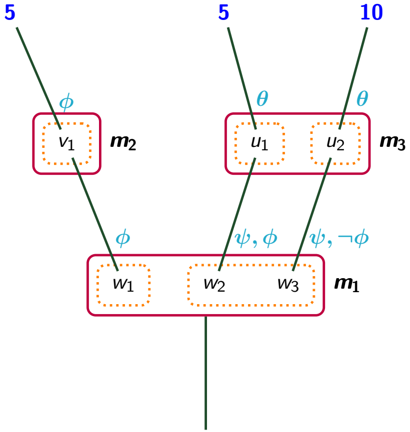

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Temporal Deontic STIT (TDS) (1/3)

Available Deontic STIT logics are all **grounded in utilitarianism**:

- **However**, each utility function comes with its own disadvantages
- **Problems**: implicit **temporal features** embedded in utilities
  - None of the Deontic STIT logics is explicitly temporal (!)

Our solution

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NB. $[Ag]$ = 'grand coalition' ⇒ 'No Choice between Undivided Histories'

2. Merge **axiom systems** of [Lorini13] and [Murakami05]

3. We use **temporal frames** [Lorini13] and extend with novel **relational characterization of obligation** operators $\otimes_i$

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Temporal Deontic STIT is **complete** w.r.t. the class of TDS frames

- Canonical models w.r.t. **irreflexive** MCSs [Gabbayetal94]

Theorem 2

Every Temporal Deontic STIT model can be **truth-preservingly transformed** into a Temporal Utilitarian STIT model

Corollary 1

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NB. Kripke frames are modular and suitable for labelled calculi

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1. Proofs formalized as **mathematical objects** in their own right
2. Proofs are analyzed and investigated using mathematical techniques
3. Offers constructive approach to **studying properties of logics**

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Analytic Calculi:

1. Calculi where **proof-search** proceeds by stepwise **decomposition** of formula to be proven
2. Useful for developing **automated reasoning** methods

Calculi for STIT logics 1/8

Our Strategy (in [LyonBerkel19]):

1. **Focus**: multi-agent STIT logic with limited choices
2. **Provide**: labelled sequent calculi for this class of logics
3. **Show**: elimination of structural rules via **refinement** procedures using **propagation rules**
4. **Show**: the resulting calculi are suitable for terminating proof-search

Calculi for STIT Logics 2/8

Multi-agent STIT language (basic)

$$\phi ::= p \mid \overline{p} \mid \phi \wedge \phi \mid \phi \vee \phi \mid \Box\phi \mid \Diamond\phi \mid [i]\phi \mid \langle i \rangle\phi$$

for m-many agents $i \in Ag$

- We use NNF to reduce the amount of sequent rules later on
- Classical negation, conjunction, and disjunction
- Settledness $\Box$ and Choice $[i]$ (for every agent)

Calculi for STIT Logics 3/8

Multi-agent STIT logic with n-limited choices: Ldm_n^m

- S5 for $\Box$ and $S5$ for $[i]$ (recall: equivalence classes!)
- Bridging moments and choices: $\Box\phi \to [i]\phi$

Calculi for STIT Logics 3/8

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- **Independence of agents** (choice-consistency):

$$\Diamond[1]\phi_1 \wedge \cdots \wedge \Diamond[n]\phi_n \rightarrow \Diamond([1]\phi_1 \wedge \cdots \wedge [n]\phi_n)$$

Calculi for STIT Logics 3/8

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$$\Diamond[i]\phi_1 \wedge \Diamond(\overline{\phi}_1 \wedge [i]\phi_2) \wedge \cdots \wedge \Diamond(\overline{\phi}_1 \wedge \cdots \wedge \overline{\phi}_{n-1} \wedge [i]\phi_n) \rightarrow \phi_1 \vee \cdots \vee \phi_n$$

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Theorem [Xu94]

Ldm_n^m is sound and complete with respect to the class of Ldm_n^m frames

Calculi for STIT Logics 4/8

Proof calculi $G3Ldm_n^m$ with **labelled sequents** [Negri05]:

- Labelled Sequents: $\mathcal{R}_i xy, x : \phi$

Calculi for STIT Logics 4/8

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$$\frac{}{\Gamma, w : p, w : \overline{p}}\ (\mathsf{id})$$

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- **Logical rules**, such as:

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Calculi for STIT Logics 4/8

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- **Structural rules**, such as:

$$\frac{\mathcal{R}_i ww, \Gamma}{\Gamma} \; (\mathrm{refl}_{[i]}) \qquad \frac{\mathcal{R}_i wu, \mathcal{R}_i wv, \mathcal{R}_i uv, \Gamma}{\mathcal{R}_i wu, \mathcal{R}_i wv, \Gamma} \; (\mathrm{eucl}_{[i]})$$

$$\frac{\mathcal{R}_1 u_1 v, ..., \mathcal{R}_n u_n v, \Gamma}{\Gamma} \; (\mathrm{IOA})^*$$

Calculi for STIT Logics 5/8

Theorem

The $G3Ldm_n^m$ calculi have the usual **properties** such as [Negri05]:

- Admissibility of contraction and cut

$G3Ldm_n^m$ is sound and complete relative to Ldm_n^m

What about **automated proof-search**?

We proceed via **refinement methods** [TiuIanovskiGoré12]:

1. Reduces structure in sequent (via adding propagation rules)
2. Enables more compact proofs (via eliminating structural rules)
3. Facilitates termination (loop detection!)

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Calculi for STIT Logics 6/8

Step 1: We extend $G3Ldm_n^m$ by adding **propagation rules**:

$$\frac{\mathcal{R}, w : \langle i \rangle \phi, u : \phi, \Gamma}{\mathcal{R}, w : \langle i \rangle \phi, \Gamma} \; (\mathsf{Pr}_i)^{\dagger}$$

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Two side conditions (†):

1. **Transform** sequent of the prem/concl of the rule to an **automaton**
2. Certain **strings** in the automaton allow for **correct application**

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$$\mathcal{R}_1 wu, \mathcal{R}_2 uv, \mathcal{R}_1 vz \; = \; \begin{tikzcd} \boxed{w} \arrow[r, bend right, dotted, "\langle 1 \rangle"'] & u \arrow[l, bend right, dotted, "\langle 1 \rangle"'] \arrow[r, bend right, dotted, "\langle 2 \rangle"'] & v \arrow[l, bend right, dotted, "\langle 2 \rangle"'] \arrow[r, bend right, dotted, "\langle 1 \rangle"'] & z \arrow[l, bend right, dotted, "\langle 1 \rangle"'] \end{tikzcd}$$

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Calculi for STIT Logics 6/8

Step 2: We **refine** the extended calculi by showing the following:

Theorem

Structural rules for $(\mathsf{eucl}_{[i]})$ and $(\mathsf{refl}_{[i]})$ are **eliminable** in the light of **propagation rules**

i.e. propagation rules preserve relational atoms $\mathcal{R}$ (!)

Propagation Rules: $$\frac{\mathcal{R}, w : \langle i \rangle \phi, u : \phi, \Gamma}{\mathcal{R}, w : \langle i \rangle \phi, \Gamma} \; (\mathsf{Pr}_i)^{\dagger}$$

Structural Rules: $$\frac{\mathcal{R}_i ww, \Gamma}{\Gamma} \; (\mathsf{refl}_{[i]}) \qquad \frac{\mathcal{R}_i wu, \mathcal{R}_i wv, \mathcal{R}_i uv, \Gamma}{\mathcal{R}_i wu, \mathcal{R}_i wv, \Gamma} \; (\mathsf{eucl}_{[i]})$$

Calculi for STIT Logics 7/8

Main results for the class of **refined multi-agent** STIT calculi:

1. Calculi are Sound and Complete
2. The Cut (left) and Contraction (right) rules are admissible

$$\frac{\Gamma, x : \phi \qquad \Gamma, x : \overline{\phi}}{\Gamma} \qquad\qquad \frac{\Gamma, x : \phi, x : \phi}{\Gamma, x : \phi}$$

Main results for the class of **refined single-agent** STIT calculi:

1. **Proof-search algorithms** for the class of Ldm^1_n
   - Correct and terminating
2. Automated (finite) **counter-model** extracting via failed proof-search
3. Every derivable formula is derivable only using **forest-like** sequents

Calculi for STIT Logics 7/8

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Outline of this talk

Conclusions and Future Work

Conclusions

1. Sound and complete Temporal Deontic STIT logic
2. Sequent-style calculi for STIT logics (Ldm_n^m, $TSTIT$, $XSTIT$, TDS)
3. Proof-theoretic decidability and automated counter-model construction for $G3Ldm_n^1$

Future Work

4. Obtain automatic proof-search for Multi-Agent STIT ✓
5. Obtain automatic proof-search for Deontic STIT Logic
6. Decidability problems for extensions (e.g. Temporal Deontic STIT)
7. Apply the calculi to the analysis of legal and moral reasoning

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Fin. Thanks!